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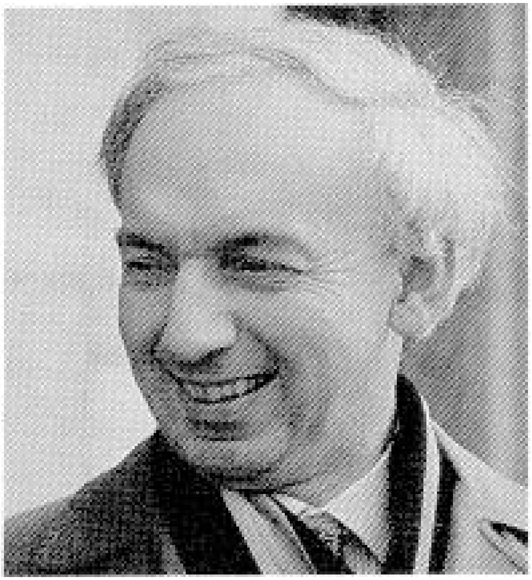
Work for the Hairdressers: The Production of de Prony's Logarithmic and Trigonometric Tables

I. GRATTAN-GUINNESS*

Author's Address: Middlesex Polytechnic at Enfield, Middlesex EN3 4SF, England



*I. **Grattan-Guinness** *has both the doctorate (Ph.D.) and higher doctorate (D.Sc.) in the history of science at London University. He is currently Reader in Mathematics at Middlesex Polytechnic, England. He was editor of the history of science journal* ***Annals of Science*** *from 1974* to 1981, and is founder-editor of the journal **History and Philosophy of Logic**, launched in 1980. His **Convolutions in French Mathematics, 1800-1840: From the calculus and mechanics to mathematical analysis and mathematical physics** is to appear in three volumes, 1990, Basel and Berlin. He is editing a sub- stantial and comprehensive **Encyclopaedia of the History and**

***Philosophy of the Mathematical Sciences**, which is to be published in 1992. He is also working on the history of mathematical logic and the foundations of mathematics from 1870 to 1930.*

A summary account is given of the large set of logarithmic and trigonometric tables produced at the end of the 18th century *under the direction of Gaspard Riche de Prony. Although they were completed in 1801, their size made publication a costly task; and it was never done, despite the fact that printing was started more than once and various efforts were made over the years to find finance. Eventually a reduced* *edition of some tables appeared in 1891. In addition to the history some details are given of the mathematical methods used in the compilation of the tables. An appendix considers the possible influence of this project upon Babbage's ideas on computers and tables.*

Categories and Subject Descriptors:

K.2. **[Computing Milieux]:**

History of Computing-people, theory.

G. 1.1. **[Mathematics of Computing]:**

Numerical analysis, Interpolation.

General Terms: Theory.

Additional Terms: Logarithmic tables,
Trigonometric tables,
Gaspard Riche de Prony,
Charles Babbage.

The Career of de Prony: Education and Mathematics for the Engineer

The great tables project of de Prony is often mentioned in books and articles on the history of computing and (occasionally) of numerical mathematics; but the statements made are often in passing and sometimes inaccurate, for no connected account seems to be available. The purpose of this article is to pull together information from various sources on this remarkable effort.

Gaspard Clair François Marie Riche de Prony (1755-1839) (Figure 1) was born in the Beaujolais region of southern France to the Riche family. They owned land at a location nearby called “Prony,” and he added this name to his own. I shall refer to him as “de Prony,” in conformity with his own preferred form in his many publications; but his younger brother Claude used only the original surname during his career as a distinguished naturalist.

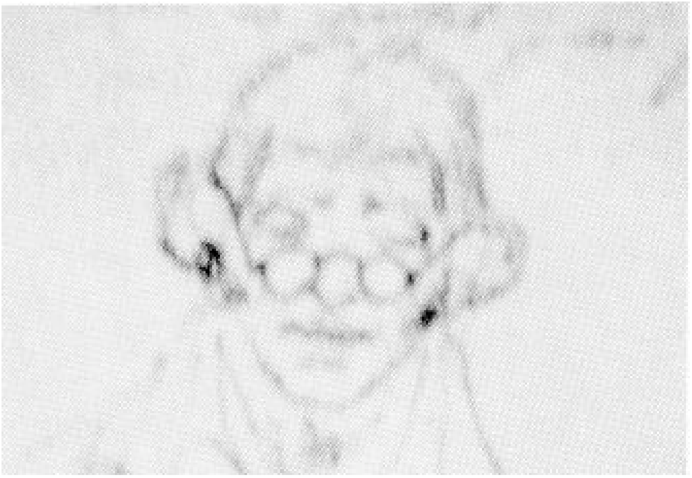


Figure 1. Sketch of de Prony teaching at the ***Ecole Poly-technique*** in the early 1800s just after the tables were completed. It was doodled in his notebooks by a student called Atthalin, who entered the school in 1802. It is the only picture of de Prony of that

time of which I am aware; other sketches in the notebook (of Monge, for example) show that Atthalin had a considerable artistic ability.

After studying at the Ecole des Ponts et Chaussées in Paris in the late 1770s de Prony joined the associated engineering corps but maintained connections with the school. In the reforms of educational institutions following the Revolution of 1789 he was appointed professor of analysis, with Lagrange, at the newly founded Ecole Polytechnique from 1794, transferring to a post of graduation examiner in 1816. From 1798 he was also made Director of the Ecole des Ponts et Chaussées, another post which he retained until the end of his life. He was elected in 1795 to the new classe des sciences mathématiques et physiques of the Institut de France, the learned body which replaced the old Academic des Sciences, and he retained this position in 1816 when that institution was restored after the fall of Napoleon. In 1801 he was appointed to a supplementary position at the Bureau des Longitudes. He held all these posts throughout his life, and also some subsidiary ones at various times, such as the directorship of a small Ecole de Géographes which he ran from 1795 until its closure in

1803. Such was the style of professional life in the Paris of his time: the so-called "cumul" system, where a scientist held several posts at once.

de Prony is remembered today only for his tables, and for a simple dynamometer (the "de Prony brake") which he introduced in the late 1810s [1]; but in his lifetime he was very well known, for his numerous activities in engineering practice and education, and in many related areas of science. He wrote a few treatises, a large number of papers, and also several editions of his lectures on mechanics at the Ecole

Polytechnique [2]. He brought a philosophy to much of his work that bore strongly upon his tables project: the desire to render theories in mathematical form, and in versions which were amenable to numerical calculation and accessible to observational data. For example, at the Ecole Polytechnique he included an optional but substantial course on difference equations and related topics, publishing it at length in the school's journal as *Suite des cours d'analyse* (de Prony 1796). He formed a stark contrast with his colleague professor Lagrange, who preferred a theoretical algebraic approach to the calculus of his own devising (although the “Lagrange interpolation formula” belongs to this time also). I shall return to this contrast in the appendicial section on Babbage.

On the History of the Tables Project

In 1790, when he was in his 36th year, citizen de Prony was appointed chief engineer to the departement of Oriental Pyrénées [3] but he announced his desire to stay in Paris [4]. This predicament may have inspired him to launch the tables project, which was partly born out by the reform of weights and measures then being undertaken [5]. Perhaps opportunely, therefore, early in the new decade he set up a Bureau de Cadastre in Paris, to prepare a detailed map of France to facilitate the accurate measurement of property as a basis of taxation [6]. In connection with this plan, it was decided that a very large set of tables would be prepared.

From the mid-1790s the Bureau seems to have been housed with de Prony's Ecole de Géographes, and was also responsible for the calculations of the navigational tables required by the (newly formed) Bureau des Longitudes for publication in their annual journal *Connaissance des temps*. The mathematical details of the tables project will be surveyed in the next section: the chronology is described here, as well as it can be traced.

This qualifier needs some explanation. Despite the fame (or notoriety) of the project, connected information is hard to find, and some sources contradict details given in others [7]. I have not been able to find even the address of the Bureau de Cadastre, nor have I traced its archive (if it still exists). No useful information exists on the organization of the work-room, the full personnel, or the budgets for the project; and apparently the waste sheets used for producing or checking the tables were discarded after use. Given these provisos, here is brief outline of the history of the tables:

de Prony devised the project explicitly following the principles of the division of labor, which had been laid down by Adam Smith in *A Treatise on the Wealth of Nations* of 1776. As he later recalled, I came across the chapter where the author treats of the division of work [travail]; citing, as an example of the great advantages of this method, the manufacture of pins. I conceived all of a sudden the idea of applying the same method to the immense work with which I had been burdened, and to manufacture logarithms as one manufactures pins. I have reason to believe that I had already been prepared for this conception by certain parts of mathematical analysis, on which I was then lecturing at the Ecole Polytechnique [8].

de Prony gave some details of the project in a “Notice” (de Prony 1801) read to the *classe* soon after it was finished. The first section had contain-ed a handful of mathematicians, including A. M. Legendre, C. A. Prieur de la Cote d’Or, and L. Carnot (the former two also being involved with the reform of weights and measures, and latter two also acting as influential political figures); they chose the mathematical formulae to be used for calculation and checking, and also considered the choice of initial values of the numbers or angles, the numbers of decimal places to be adopted in each table, and so on. The second *section* comprised several “Calculators,” including the mathematicians A. M. Parseval (of the well-known formula) and J. G. Garnier, who determined the values, and the differences of various orders, that needed to be calculated. They also prepared a page of tables for the numerical work by laying out the columns of the chosen values and the first row of entries, and preparing the instructions on the preparation of the remaining entries on the page. These calculations were done by the third section, a large team of between 60 and 80 assistants [9]. Many of these workers were unemployed hair-dressers: one of the most hated symbols of the ancien regime was the hairstyles of the aristocracy, and the obligatory reduction of coiffure “as the geometers say, to its most simplest expression” left the hairdressing trade in a severe state of recession. Thus these artists were converted into elementary arithmeticians, executing only additions and subtractions [10].

When a page was completed, it was returned to the second section, to check the figures using formulae chosen by the first section. The project was run twice, in that two sets of each table were produced from different equations, so that each set could be controlled against the other." By 1794, 700 results were being produced each day (Lalande 1803, pp. 743-744).

After completion of the project in 1801, several of the calculators were transferred to the Bureau des Longitudes to carry on similar work there (Bigourdan 1928, pp. A25-A28). In addition, some of the equipment that had been used (presumably on the preparation of the map) went to the Depot Général de la Guerre, a military organization concerned with geodesy and cartography (Berthaut 1902). At that time de Prony read his “Notice” to the classe, and the task was set to find money to finance the printing.

One of de Prony’s supporters was the astronomer Jean Baptiste Joseph Delambre (1749-1822); he completed by 1801 a much smaller set of logarithmic and trigonometric tables initiated by the mathematician and instrument designer J. C. Borda, who had died in 1799 (Borda and Delambre 1801). Delambre himself was much concerned with the compilation of tables (in his case, usually astronomical ones); and he was to be an influential aid to de Prony, for in 1803 he was appointed founder secrétaire perpétuel of the classe for the (so-called) “mathematical” sciences, having served previously in the rotating post of secrétaire. He gave the project some advertising in one of his regular reports of the activities of the classe (Delambre 1801a, pp. 195-198) and also in a separate report (Delambre 1801b) annexed to de Prony's "Notice." This piece and de Prony's "Notice" were reprinted in the mémoires of the classe in 1804, whereupon de Prony added an historical note (de Prony 1804) on the errors found in previous tables.

The publishers Firmin Didot were charged with the task of printing, and estimated that a volume of 1200 large pages would result, together with the introduction, at a cost of 80,000 francs; apparently about

500 pages were on proof by 1802. But financial crises in France began to intervene, and printing stopped. Nevertheless, hopes remained alive for many years to complete the work. Firmin Didot announced the volume(s) in some of their catalogues of the 1810s. Legendre published some entries

from the logarithmic tables in his treatises on transcendental and elliptic integrals [12]. At the end of that decade, and following the fall of Napoleon, collaboration was sought with the British government for financial support, especially through the offices of the physician Sir Charles Blagden; but

Blagden's death in 1820, and the British desire to have the tables converted from the original centesimal system of angular division to the sexagesimal system, scuppered these hopes [13]. de Prony announced the possible collaboration to the Académie des Sciences (as it had rebecome) in a pamphlet (de Prony 1820b), but in a companion piece (de Prony 1820a) he hoped that the “unique monument” could be kept in its original form. In a public lecture (de Prony 1824) to the Académie, he revived interest to the extent that printing seems to have begun again; but once again it ceased, and Firmin Didot formally returned their set to the Bureau des Longitudes in 1833. Only a few printed pages seem to have survived [14]. After de Prony's death 1839, hopes still remained for printing (according to the obituary Dupin (1840, p. 338)); but they were not realized.

However, both sets of the manuscript tables are still preserved in Paris. The one just mentioned is held in the library of the Paris Observatoire. de Prony himself retained the other set; but his niece and heir did not include it in his manuscripts that she sent to the Ecole des Ponts et Chaussées. It was found in 1858, in the possession of the niece's daughter and son-in-law, by Pierre Alexandre Francisque Lefort(-Latour) (1809-1878), in the course of making an extensive study of the other set in the Observatoire. Lefort was a graduate of the Ecole Polytechnique and the Ecole des

Ponts et Chaussées (during the time that de Prony had been graduation examiner and director respectively), and then a career engineer in the Corps des Ponts et Chaussées. His grandfather-in-law J. B. Biot reported his finding to the Académie des Sciences (Biot 1858), and this set was placed in the library of the Bibliothèque de l'Institut (manuscripts 1496-1514). Biot made a statement to the Académie (Lefort 1858a), and sent a more detailed report (Lefort 1858b) to the Observatoire, where the other set was still retained.

Each set comprises 19 volumes, of which the last one consists of an account of the methods of calculation (and a few basic tables). The others are large format tomes, often of 251 folios each (containing the tables for a run of 25,100 values of the numbers or angles). The first eight volumes contain logarithms of numbers from 1 to 200,000 calculated to 14 decimal places; then follows (not necessarily in this order) a volume of sines of parts of the radius, three volumes each of sines and of tangents (including their logarithms for some ranges of the argument), one volume each of the ratios of arcs to sines and of arcs to tangents, and the product of sines with cosines. In fact, only the Institut set contains this last volume: presumably the other set had a copy also, but it is now lost [15]. Further, the last volume in the Institut was originally with its brother in de Prony's family; it was transferred from the *Observatoire* some time after 1858, and gives a less complete account.

When de Prony's set had been reported to the Académie des Sciences in 1858, hopes had again been expressed that printing could be achieved (Biot 1858). The collation of the two sets was completed in, 1862 by officers of the Dépôt Général de la Guerre [16]. In his discussion of the tables, Lefort was doubtful of the value of publishing the tables, but thought that a rounded-off version to eight places was desirable (Lefort 1858b, p. 146) [17].

Either by accident or influence, 30 years later the Service Géographique de l'Armée of the Dépôt published just such an edition, as a replacement to the Borda-Delambre tables, which went only to six or seven places. This edition comprised 640 (printed) large quarto pages, of the (truncated) logarithmic, and log sine, cosine, tangent, and cotangent tables, with the latter still rendered in centesimal angular division (Service Géographique de l'Armée 1891). The year of publication, 1891, was close to the centenary of the project; but this detail was not mentioned in the preface written by the Director of the Service Géographique de l'Armée, and may not have been noticed [18]. In fact, this volume did not even mention de Prony in its title (so that its existence has understandably been overlooked by historians); and in the short preface the director reported that the

Borda-Delambre tables were practically sold out, and mentioned the need for tables which go beyond seven places.

The rounded-off tables were transcribed from the Observatoire set and the proofs checked twice against the original. It seems that few errors were found. So congratulations are due to de Prony for his design of the project and hats off to the hairdressers, too, at least for the early places of the long computations.

Notes on de Prony’s Mathematical Methods

Lefort worked for several months with the Observatoire set, and his article (Lefort 1858b) contains the most substantial account of the methods used to prepare the tables. Here I provide a representative selection of the methods, referring the reader for further details to his article, to de Prony's final descriptive volumes in the sets of tables (his published articles did not contain much technical information), and to a discussion of the mid-1870s held between Lefort and the Scottish mathematician

Edward Sang (1805-1890), who had recently produced some large logarithmic tables of his own and was critical of some of de Prony's procedures (Sang 1874, 1875; Lefort 1875).

As de Prony stated (de Prony 1801), his design of the tables was like an inverse to normal methods. Instead of building up the final values from differences of various orders and by interpolative formulae, the exact values of $f(x)$ were calculated for some initial values x from a known formula (usually a series expansion), and the differences pertaining to succeeding values

$(x+h)$ were calculated and inserted into an adopted formula to produce the required values of $f(x+h)$.

A main motive for this strategy was to avoid multiplication and division and to reduce the calculations to sums and (especially) differences, which the hairdressers could fairly be expected to handle. In consequence there was need for formulae expressing expansions in terms of differences of various orders. One of these was a theorem due to Euler,

$$u\,(x+nh) - u(x) = \sum^{n}_{r=1} {}_{n}C_{r}\, \Delta^{r} u(x), \qquad (1)$$

where the forward difference was defined as usual:

$$\Delta u(x) = u(x + h) - u(x). \quad (2)$$

An iterative formula on orders of differences on neighbouring values of function, due to the 17th century astronomer G. Mouton, played an important role:

$$\Delta^{n-1} u(x) = \Delta^{n-1} u(x - h) + \Delta^{r} u(x - h) \quad (3)$$

(in terms of the definition of Δ u(x) in (2)). The names of the required differences were laid out in columns on the page by the second section for the hairdressers to calculate as per instructions in the body of the page. These computations proceeded up to the column of differences (prepared by the second section), which contained (almost) equal figures in each row, which of course the process stopped.

Among special formulae used, the logarithms of prime numbers were calculated from an expansion which I write as

$$2 \log x = \log (x + 1) + \log (x - 1)$$

$$+ (2 \log e) \Sigma^{\infty}_{r=0} [z^{2r+1}/(2r + 1)] \qquad (4)$$

where $z = (2x^2 - 1)^{-1}$; (5)

further terms were deliverable via expansions such as

$$\log (y + x) - \log (y - x)$$

$$= (2 \log e) \Sigma^{\infty}_{r=0} [(x/y)^{2r+1}/(2r + 1)]. \qquad (6)$$

Logarithms of compound numbers were produced from those of their prime factors by addition.

For the trigonometric tables the standard power-series expansions were used for the initial entries over 10º intervals [19], and addition formulae for sines and cosines were deployed for intermediate values (for sin(a + b), for example, where a took values in steps 0.1° and b in steps of 0.01°). For higher-order differences de Prony adopted “the extremely elegant and simple formulae proposed by the citizen Legendre, ” rather similar in form to Mouton's (3):

$$\Delta^n \sin(x) = -(2\sin \Delta x/2)^2 (\Delta^{n-1} \sin x + \Delta^{n-2} \sin x), \quad n \geq 1 \qquad (7)$$

for extreme values of x in the range, and for intermediate values

$$\Delta^n \sin x = \cos x \, \Delta^n \sin 0 + \sin x \, \Delta^n \sin 1. \qquad (8)$$

The tangent tables were devised by various formulae. Up to 50º difference formulae were deployed; then up to 94° recourse was made to

$$\tan(50^\circ + a) = 2\tan 2a + \tan(50^\circ - a); \quad (9)$$

and for the last few a power series in cot x (which would then be taking very small values) was deployed.

For log sine table, results such as

$$\log (ax/\sin ax) = \log ax - \log \sin ax, \text{ with } a > 0 \quad (10)$$

were used, with ($ax/\sin ax$) expanded in power series of ax [20]. For differences of various orders, some use was made of Lagrange's symbolic generalized version of Taylor's theorem:

$$\Delta^n u(0) = [\exp(\Delta x f'(x) - 1)]^n, \qquad (11)$$

where $u(m) = f(x + m\Delta x)$.

This formula required knowledge of the derivatives of $f(x)$: for log sin x, these were expressed as polynomials in cot x, which was small when x was close to 100º. Log tan tables were produced from differences such as

$$\Delta^n \log \tan x = \Delta^n \log \sin x - \Delta^n \log \cos x \qquad (12)$$

and various similar formulae derivable from it.

Some check formulae were used. A favored one for the sine formulae was Euler's:

$$\sin x + \sin(40^\circ - x) + \sin(80^\circ + x)$$
$$= \sin(40^\circ + x) + \sin(80^\circ - \mathrm{x}). \qquad (13)$$

An impression of pragmatic action comes over from this repertoire of techniques (which would be reinforced by a more detailed account). Part of the reason was a need to have two different sets of formulae for the two sets of tables, and part was caused by the perceived need to calculate the entries to so many decimal places (the numbers varied between 14 and

29). In the design some obvious moves were missed. For example, as Sang pointed out, de Prony did not employ the obvious relation:

$$\begin{aligned} \log_{10}(10x) &= \log_{10}10 + \log_{10}x \\ &= 1 + \log_{10}x \end{aligned} \tag{14}$$

either as a means of reducing or simplifying interpolations or as a check on the decimal parts of the logarithmic tables. Sang also criticized the strategy of rounding or truncating the differences of the highest orders (which he called a method of "vitiated differences"), on the ground that rounding or truncating errors were prematurely introduced already in the values found at the higher orders of difference and therefore cumulation could occur upon transmission back to the lower orders. Lefort showed that the possible errors obtainable from (1) were not as large as

Sang feared; but he had his own criticisms of the project, such as deploring the apparent discard of the loose sheets upon which calculations had been made before transcription of the results onto the pages, and disliking de Prony’s method of producing the final values for the log sine table. By and large, Lefort's reservations and Sang's criticisms carry weight; even though the Service Géographique de l'Armée found few errors in the tables when preparing their version for publication, the manner of compilation seems to have been corrigible.

One aspect of the tables which is not too clearly described is the effect of errors of calculation. One would presume that the errors would (or could) iterate onto entries in the vicinity of their occurrence; but they seem to be occasional, even if fairly frequent in places (and written in red ink above the black originals). In a passage which excites suspicion, Lefort reports detecting "arbitrary corrections" in places in the tables (Lefort 185813, p. 146).

What remains unexplained is the reason why de Prony chose to calculate these tables to such extraordinary numbers of decimal places in the first place. Did the taxation system really require the cadastral survey to be such an exact science, or did de Prony want to keep the threat of the Pyrénées appointment of 1790 well at bay? Delambre knew the project well, and expressed himself with careful vagueness to

Blagden in 1819 when the collaboration with the British Government was in the offing. In contrast to his own completion of the Borda tables, he opined that “These tables will not serve in the usual cases, but only in extraordinary cases” [21].

Appendix: Influences Upon Babbage

Babbage gives two occasions for the origins of his interest in the mechanical calculation of tables one from 1812 and the other from 1819 (Babbage 1864, pp. 42-43). The latter date is especially interesting, as it coincides with Blagden's involvement in the possible printing of the tables. Babbage does not refer to de Prony here, but he must have surely heard of the project; and he may have realized that mechanization was the better way to produce tables of this size. Moreover, his policy of using large numbers of decimal places, working with sequences of differences of various orders is reminiscent of de Prony (although of course it was not novel with either man). Indeed, soon afterwards, in a pamphlet of 1822 on “the application of machinery to the purpose of calculating and printing mathematical tables” –his main publication for securing governmental support for production of the Difference Engine– he gave some account of de Prony's project and noted that mechanical methods would speed up the process of calculation. Clearly he was struck by de Prony's production of the tables following an industrial process, and was hoping to imitate the process by mechanical means. In his book on manufactures, he rehearsed some of the same material on de Prony's project and on mechanical calculation (Babbage 1832, pp. 241-250).

The nature and extent of de Prony's influence on Babbage needs some exploration. It seems that Babbage had already envisioned his idea of mechanical calculation before learning of de Prony's tables; but he may well have been helped to crystallize some of his ideas, recognize difficulties, and so on, by acquaintance with that project. Babbage does not seem to have met de Prony or corresponded with him, but he examined the Observatoire set of tables during his visit to Paris in 1826 [22], and had copies made of some of de Prony's writings on the project [23].

Finally, another influence is worth recording here, although it is of a somewhat different kind. It comes from de Prony's colleague professor at the Ecole Polytechnique, Lagrange; as we saw in the first section, he preferred algebraic versions of the calculus (and indeed of all branches of mathematics). His position was based on the assumption that every function $f(x + h)$

of a real variable *x* could be expanded in a Taylor series for every value of *x* (apart from values of *x* when *f* misbehaved, such as taking an infinite value), and that the derivatives could be defined, by purely algebraic means, as the coefficients of the appropriate powers of the incremental variable *h* [24]. In the course of pursuing this line, he gave considerable impetus to the development of two new algebras, differential operators and functional equations; and these topics, especially the latter one, were to constitute the first research interest of the mathematician Babbage from the mid-1810s onwards [25].

Now one of the strengths of these algebras (and a source of reservation for their critics) was their *algorithmic power:* to handle such a function f, its doubly applied function $ff\,(=f^2)$, $f^3, \ldots,$ the inverse function(s) f^{-1}, and so on; and these features which are particularly marked in the rather wild manipulations which Babbage exhibited in his mathematical papers. Can we see Babbage as a naturally algorithmic mathematical thinker matched to a practically oriented personality? Was he, in a way, a fusion of the interests of Lagrange and de Prony? He hinted in such a concordance in a passage in his autobiography (1864, pp. 435-436):

Calculus of Functions.

This was my earliest step, and is still one to which I would willingly recur if other demands on my time permitted . . . . It is very remarkable that the Analytical Engine adapts itself with singular facility to the development and numerical working out of this vast department of analysis.

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Note

[1] The most substantial survey of de Prony's career is Bradley (1984). Walckenaer (1940) contains a series of articles on his life and work, which also feature in my

(1990), esp. passim in chs. 2, 3,5, 6, 8 and 16. His manuscripts and library both massive collections are held in the Ecole des Ponts et Chaussées in Paris.

[2] Unfortunately for the historian, de Prony also produced quite a number of short pamphlets, which are now very difficult to find. Three of them relate to the tables project.

[3] After the Revolution France was divided into regions known as "départements," each one run by a Prefet based in the chief city. Basically the system is still maintained today.de Prony had been condemned to an

engineer's post in a *département* near to the Spanish border: see the letters in the very interesting file on his career held in the Archives Nationales (Paris), F^{14} 2304^{2}.

[4] The desire to remain in Paris is an important social factor to bear in mind in considering all aspects of French history (and current affairs, indeed!).

[5] On the context of the project in French cartography of the time, see Konvitz (1987, pp.

47-62.). P. Bret (Paris) is studying these developments in depth. Herbin and Pebereau (1953) is a moderately useful history of French cadastral surveys. For the work of Borda and others on tables and on related projects of this time, see Mascart (1919, esp. pp. 580-593).

[6] For the map de Prony used a “central” projection, where the supposedly spherical earth was projected onto a tangent plane; for a country of the shape and size of France, good representations of geodesics and areas were obtained.

[7] For example, de Prony gives various dates for the date of commencement of the project: “an 2”

(1793-94) (1801, p. 49); but 1791 (1824, p. 4), and also a manuscript career document held at the Ecole des Ponts et Chaussées, ms. 1786. Compare notes [9], [15], and [18] for other examples of inconsistency.

Various other files in the Ecole des Ponts et Chaussées (see note [1]) are relevant (Konvitz

1987, pp. 165-1661, as are the minute-books of the Bureau des Longitudes. There are also files in the Archives Nationales, F^{17} 1393 and 13571, and very probably other files there (for example, F^{14} 2146 is quite useful); but the state of cataloguing is so poor that even resident Parisians despair of ever finding all the information contained on any topic.

[8] de Prony (1824), p. 4; compare his friend the educationist and novelist Maria Edgeworth on him in Colvin (1979, p. 151).

[9] This are the numbers indicated in de Prony (1801, p. 53); but according to (de Prony 1832, pp.

67-68), between 150 and 200 assistants were involved.

[10] The quotation here, and the attached information, comes from Dupin (1824, p. 23).

[11] Some of the early electronic computers produced after the Second World War also used double calculation, as a check upon reliability.

[12] Legendre (1816), table V; repeated in (Legendre 1826, pp. 260-268). He took the logarithms for all odd numbers from 1,163 to 1,501, and the prime numbers thereafter up to 10,000.

[13] News of the possible collaboration reached the newspapers (*Moniteur Universel* (1819),

1145): coming only four years after the fall of Napoleon, it was noteworthy. But I am not sure what form the conversion of the tables was intended to take. Either they were to be reformulated (presumably using some method of interpolation), or the values of the angles were to be restated in the normal sexagesimal system.

The first option was impossibly large, and the second pointless: perhaps this is why nothing happened.

[14] These pages are in the file in the Archives Nationales cited in note [4]. Lefort (1858b, p. 138) reports having seen six incomplete printed

copies of the sine tables, but he gives no provenance.

[15] In yet another characteristic inconsistency, de Prony stated (de Prony 1801, p. 54) that there were only 17 volumes of tables (presumably excluding this one); but in another reference (de Prony 1832, p. 24) he referred to 18 volumes.

[16] The last volume of the set is so annotated and signed.

[17] In fact, de Prony himself reported that during the run of the project, in response to a ministerial request, he had his bureau prepare a set of some trigonometrical tables to nine decimal places (de Prony 1824, p. 7). This set is in the archives of the *Ecole des Ponts et Chaussées* along with various other manuscript tables (Ecole des Ponts et Chaussées 1886, pp. 4-5).

[18] The short preface cites Lefort (1858b), and follows the Director in saying that the project had begun in 1794; compare note [7] on this question. Maybe it was the decision to do the trigonometric tables in centesimal

division that dates from 1794, in connection with the reform of weights and measures then under way. However, de Prony also stated that the contract between the government and the publishers had been taken out in 1794 (de Prony 1820b, p. 1, where he also gave other data on the costs of the printing).

[19] As was mentioned in the second section, the quadrant was divided centesimally. For convenience I use the usual symbol for degrees to indicate these angles.

[20] Delambre also used such formulae in his edition of Borda to obtain the so-called "Delambre tables" for (sin *x*/*x*) and (tan *x*/*x*); see Delambde (1793).

[21] This letter is quoted in Lefort (1858a, p. 999, and 1858b, p. 146). The letter is not among the Blagden papers in the archives of the Royal Society, but there are Blagden materials elsewhere.

[22] See Babbage (1827, p. vi), he does not mention the date of his visit, but reports that he made “extracts necessary to me for other calculations.”

[23] I am indebted to a referee for pointing out to me that pertinent materials are to be found in the Babbage archive at the Science Museum (London), file BAB U21, and in his correspondence held at the British Library, Add. Mss. 37182, 37183 and 37201. These files, and the French sources cited in note [7], could be used to reconstruct in some detail this aspect of Babbage's career.

[24] Lagrange's position was to be refuted in the 1820s by Cauchy (Grattan-Guinness (1990)).

[25] For a brief summary of this influence, see Grattan-Guinness (1979). M. Panteki is examining it in much more detail as part of her Ph.D. thesis in preparation. For Babbage’s work on tables, see Campbell-Kelly (1988).

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